ECS 455: Quiz 4 Solution

Semester/Year: 2/2010

Course Title: Mobile Communications

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Instructions

1. Separate into groups of no more than three persons.
2. Closed book. Closed notes.
3. Only one submission is needed for each group. Late submission will not be accepted.
4. **Do not panic.**

Evaluate the following expressions by hand. Show your calculation.

1. $\mathrm{DFT}\{[3 \quad -1]\} = [2 \quad 4]$

Let $\vec{x} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$.

By definition, $\vec{X} = \mathrm{DFT}\{\vec{x}\} = \Psi_N \times \vec{x}$

where N is the length of $\vec{x}$.

We've seen in class that

$$\Psi_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

Hence,

$$\vec{X} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 3-1 \\ 3+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

3. $\mathrm{IDFT}\{[1 \quad 0 \quad 0]\} = [\tfrac{1}{3} \quad \tfrac{1}{3} \quad \tfrac{1}{3}]$

By definition,

$$\vec{x} = \mathrm{IDFT}\{\vec{X}\} = \frac{1}{N}\Psi_N^* \vec{X} = \frac{1}{N}\Psi_N^* \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{N} \times (\text{first column of } \Psi_N^*)$$

$$= \frac{1}{3} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

Here, $N = 3$

2. $\mathrm{DFT}\{[1 \quad 0 \quad 0]\} = [1 \quad 1 \quad 1]$

First, recall the block matrix multiplication:

$$\begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_n \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \sum_{i=1}^{n} x_i \vec{a}_i$$

Therefore,

$$\begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_n \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \vec{a}_1$$

Here, $\vec{X} = \Psi_N \vec{x} = \Psi_N \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \text{first column of } \Psi_N$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Here $N = 3$

4. $\mathrm{DFT}\{[1 \quad 0 \quad 0 \quad 0 \quad 0]\} = [1 \quad 1 \quad 1 \quad 1 \quad 1]$

Use exactly the same reasoning as in Q2.

$$\vec{X} = \Psi_N \vec{x} = \Psi_N \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 1^{st} \text{ column of } \Psi_N$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Here, $N = 5$